

Answers Sheet

Circle Theorem

1.

$$OBA = 90^\circ$$

$$\begin{aligned} BAO &= 180 - 90 - 74 \\ &= \mathbf{16^\circ} \end{aligned}$$

Tangent meets radius at 90°

Angles in a triangle add to 180°

2.

i) 32°

ii) The angle at the circumference is half the angle at the center.

3.

$$OQP \text{ and } ORP = 90^\circ$$

Tangent meets radius at 90°

Angles in a quadrilateral add to 360°

$$= 360 - 90 - 90 - 39$$

$$= \mathbf{141^\circ}$$

4.

i) 50°

ii) Angles in the same segment are equal.

5.

i) $= 180 - 84$

$$= \mathbf{96^\circ}$$

ii) Opposite angles in a cyclic quadrilateral add to 180°

6.

i) $OLM = 45^\circ$ Angles at base of isosceles are equal.

$$= 180 - 45 - 45$$

$$= 90^\circ$$

ii) Angles at the base of an isosceles triangle are equal.

Angles in a triangle add to 180°

7.

IJK is an isosceles triangle, 2 tangents from the same point are equal.

$$\begin{aligned} \text{KJI} &= (180 - 42) / 2 \\ &= 69^\circ \end{aligned}$$

Angles at the base of an isosceles triangle are equal.

$$\text{OIJ} = 90^\circ \quad \text{Tangents meet radius at } 90^\circ$$

$$\text{OIK} = 90 - 69 = 21^\circ$$

8.

a) $\text{OCD} = 90^\circ$ Tangent meets radius at 90°
 $= 90 - 75$
 $= 15^\circ$

b) $\text{ABC} = 75^\circ$ Alternative segment theorem.

9.

Triangle XOZ = Triangle ZOY all sides are equal

$$\text{OXZ and OYZ} = 90^\circ \quad \text{Tangent meets radius at } 90^\circ$$

$$\begin{aligned} \text{XOZ and YOZ} &= 180 - 90 - 24 \\ &= 66^\circ \end{aligned}$$

$$\text{XOY} = 2 \times 66 = 132^\circ$$

$$\begin{aligned} \text{XWY} &= 132 / 2 \\ &= 66^\circ \end{aligned}$$

Angle at circumference is half the angle at the center.

10.

$$\text{OPR} = 30^\circ$$

Angles at the base of an isosceles triangle are equal.

$$\text{ORQ} = 90^\circ \quad \text{Tangent meets radius at } 90^\circ.$$

$$\begin{aligned} \text{PQR} &= 180 - 90 - 30 - 30 \\ &= 30^\circ \quad \text{Angles in a triangle add to } 180^\circ \end{aligned}$$

11.

$$\begin{aligned} \text{BAC} &= 180 - 74 - 62 \\ &= 44^\circ \end{aligned}$$

$$x = 44^\circ \quad \text{Alternative segment theorem.}$$

12.

$$\begin{aligned} FEG &= 62^\circ && \text{Alternative segment theorem} \\ EGF &= 62^\circ && \text{Angles at the base of an isosceles triangle are equal.} \\ EFG &= 180 - 62 - 62 \\ &= 56^\circ && \text{Angles in a triangle add to } 180^\circ \\ EOG &= 2 \times 56 \\ &= 112^\circ && \text{Angles at center is twice angle at circumference.} \end{aligned}$$

13.

$$\begin{aligned} ABC &= 180 - 120 \\ &= 60^\circ && \text{Opposite angles in a cyclic quadrilateral add to } 180^\circ \\ AOC &= 2 \times 60 \\ &= 120^\circ && \text{Angles at center is twice angle at circumference.} \end{aligned}$$

14.

$$\begin{aligned} BAO &= 28^\circ && \text{Angles at base of isosceles triangle are equal.} \\ BAC &= 54^\circ && \text{Alternative segment theorem} \\ OAC &= 54 - 28 = 26^\circ \\ ACO &= 26^\circ && \text{Angles at base of an isosceles triangle are equal.} \end{aligned}$$

15.

$$\begin{aligned} OXZ &= 90^\circ && \text{Tangent meets radius at } 90^\circ \\ 9^2 + 18^2 &= OZ^2 \\ 405 &= OZ^2 \\ OZ &= 20 \text{ cm} \\ &(\text{approx..}) \\ YZ &= 20 - 9 \\ &= 11 \text{ cm} \\ \text{Radius} &= 9 \text{ cm} \end{aligned}$$

16.

$$\begin{aligned} ACB &= 90^\circ && \text{Angle in a semi-circle is } 90^\circ \\ 4x + 5x + 90 &= 180 \\ 9x &= 90 \\ x &= 10 \end{aligned}$$

17.

$$\text{TSV} = (1/2)x$$

$$\text{TUV} = 180 - (1/2)x$$

Angle at circumference is half angle at center.

Opposite angles in a cyclic quadrilateral add to 180°

18.

$$\text{KLM} = x \quad \text{Alternative segment theorem.}$$

$$\text{KML} = (180 - x) / 2 \quad \text{Angles at base of an isosceles triangle are equal.}$$

$$\text{OMK} = 90 - x$$

$$\text{OMI} = 90^\circ \quad \text{Tangent meets radius at } 90^\circ$$

$$\text{OML} = ((180 - x) / 2) - (90 - x)$$

$$= 90 - (1/2)x - 90 + x$$

$$= (1/2)x$$
