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| Surname | Centre Number | Candidate Number |
| Other Names | | 0 |



GCSE

C300UA0-1



A17-C300UA0-1



MATHEMATICS – Component 1
Non-Calculator Mathematics
HIGHER TIER

THURSDAY, 2 NOVEMBER 2017

– MORNING

2 hours 15 minutes

ADDITIONAL MATERIALS

The use of a calculator is not permitted in this examination.
A ruler, protractor and a pair of compasses may be required.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.
You may use a pencil for graphs and diagrams only.
Write your name, centre number and candidate number in the spaces at the top of this page.
Answer **all** the questions in the spaces provided.
If you run out of space, use the continuation page at the back of the booklet, taking care to number the question(s) correctly.
Take π as 3.14.

INFORMATION FOR CANDIDATES

You should give details of your method of solution when appropriate.
Unless stated, diagrams are not drawn to scale.
Scale drawing solutions will not be acceptable where you are asked to calculate.
The number of marks is given in brackets at the end of each question or part-question.
You are reminded of the need for good English and orderly, clear presentation in your answers.

| For Examiner's use only | | |
|-------------------------|--------------|--------------|
| Question | Maximum Mark | Mark Awarded |
| 1. | 8 | |
| 2. | 5 | |
| 3. | 4 | |
| 4. | 2 | |
| 5. | 4 | |
| 6. | 5 | |
| 7. | 5 | |
| 8.(a) | 3 | |
| 8.(b)(c) | 5 | |
| 9. | 5 | |
| 10. | 6 | |
| 11. | 7 | |
| 12. | 5 | |
| 13. | 5 | |
| 14. | 4 | |
| 15. | 7 | |
| 16. | 5 | |
| 17. | 6 | |
| 18. | 7 | |
| 19. | 4 | |
| 20. | 5 | |
| 21. | 8 | |
| 22. | 5 | |
| Total | 120 | |

C300UA01
01

Formula list

Area and volume formulae

Where r is the radius of the sphere or cone, l is the slant height of a cone and h is the perpendicular height of a cone:

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

Kinematics formulae

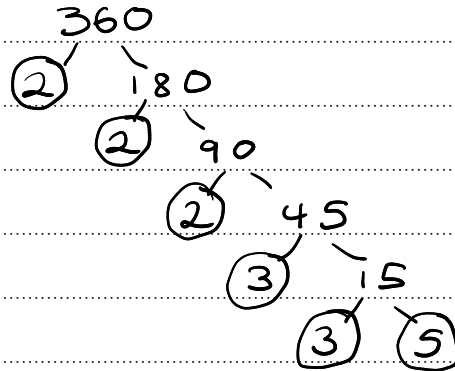
Where a is constant acceleration, u is initial velocity, v is final velocity, s is displacement from the position when $t = 0$ and t is time taken:

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

1. (a) Write 360 as a product of prime factors using index notation. [3]



$$2 \times 2 \times 2 \times 3 \times 3 \times 5$$

$$\underline{2^3 \times 3^2 \times 5}$$

- (b) In index notation, $315 = 3^2 \times 5 \times 7$.

Find the highest common factor of 315 and 360. [2]

$$3^2 \times 5$$

$$9 \times 5$$

$$\underline{\underline{45}}$$

- (c) (i) Write 5.4×10^{-4} in decimal notation. [1]

0.00054

- (ii) Find the value of $\frac{1.6 \times 10^7}{2 \times 10^2}$.

Give your answer in standard form. [2]

$$0.8 \times 10^5$$

$$8 \times 10^4$$

2. (a) Tina is carrying out a survey to find out how people use their mobile phones.

- (i) Here is a question from her survey.

What do you use your mobile phone to do?
Tick (✓) one box.

Text ☐ Call ☐ Take a photo ☐

State one criticism of this question.

[1]

- There are other things people do on mobile phones (eg play games, internet...)
- or • people may want to tick more than one of these

- (ii) Here is a different question from her survey.

How often do you use your mobile phone?
Tick (✓) one box.

All the time ☐ A lot ☐ Not much ☐ Never ☐

State one criticism of this question.

[1]

people will have different ideas of what "not much" and "a lot" mean.

- (b) Tina surveyed 205 students about the cost of their monthly phone bills. The table shows this information.

| | Number of students | Lowest bill | Mean bill | Highest bill |
|------------------|--------------------|-------------|-----------|--------------|
| Pay-as-you-go | 100 | £5 | £12.75 | £70 |
| SIM only | 100 | £15 | £16.25 | £18 |
| Monthly contract | 5 | £28 | £40 | £60 |

- (i) Comment on how reliable the data about Monthly contracts are likely to be. [1]

not very reliable due to small sample size.

- (ii) Using the data in the table, Tina compares the cost of Pay-as-you-go with the cost of SIM only. Tina says that students who use Pay-as-you-go have both the lowest and highest bills.

Make further comments to explain why Tina may think

- SIM only is a better deal,
- Pay-as-you-go is a better deal.

[2]

Complete each of the following statements.

SIM only could be a better deal because the range of bills is lower.

Pay-as-you-go could be a better deal because the average (mean) cost is lower.

3. $\mathbf{p} = \begin{pmatrix} 5 \\ 4 \end{pmatrix}$ and $\mathbf{q} = \begin{pmatrix} 1 \\ -1.5 \end{pmatrix}$

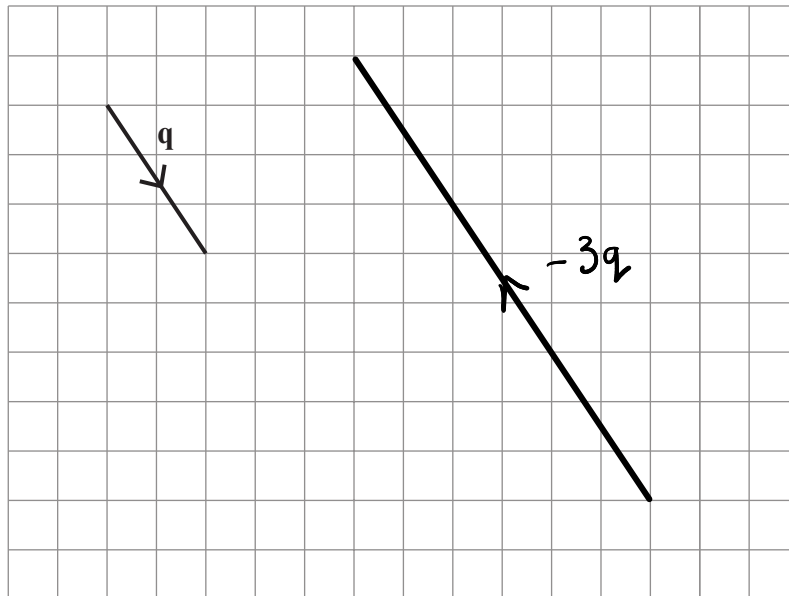
$$2\mathbf{p} = \begin{pmatrix} 10 \\ 8 \end{pmatrix}$$

(a) Work out the column vector $2\mathbf{p} - \mathbf{q}$.

[2]

$$\begin{pmatrix} 10 \\ 8 \end{pmatrix} - \begin{pmatrix} 1 \\ -1.5 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 9 \\ 9.5 \end{pmatrix}}}$$

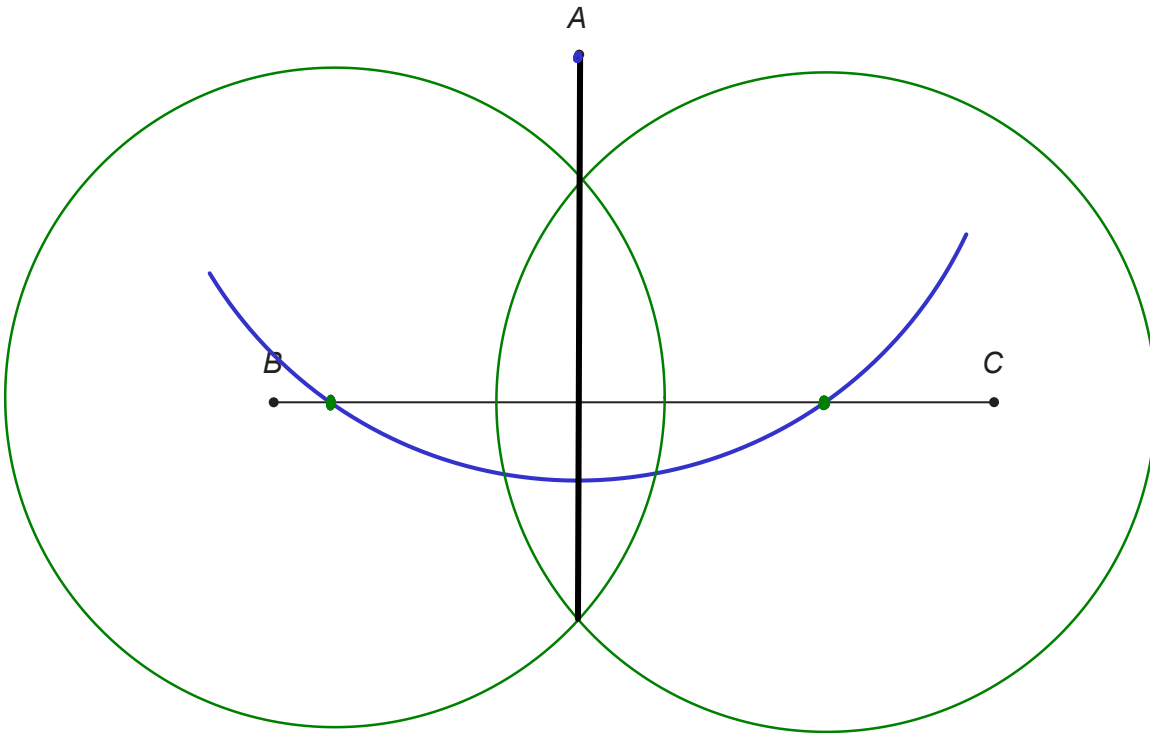
(b) The grid shows the vector \mathbf{q} .



On the same grid, draw the vector $-3\mathbf{q}$.

[2]

4. Using a ruler and a pair of compasses, construct the perpendicular from A to the line BC . You must show your construction arcs. [2]



5. (a) Expand and simplify $(x - 5)(x + 2)$. [2]

$$x^2 + 2x - 5x - 10$$

$$\underline{\underline{x^2 - 3x - 10}}$$

- (b) Simplify $3a^2 \times 6a^{-1}$. [2]

$$\underline{\underline{18a}}$$

6. (a) The variables x and y are connected by the equation $y = \frac{25}{x}$.

(i) Circle the correct statement.

[1]

x is inversely proportional to 25

x is directly proportional to y

y is inversely proportional to x

y is directly proportional to x

(ii) Find the value of x when $y = 100$.

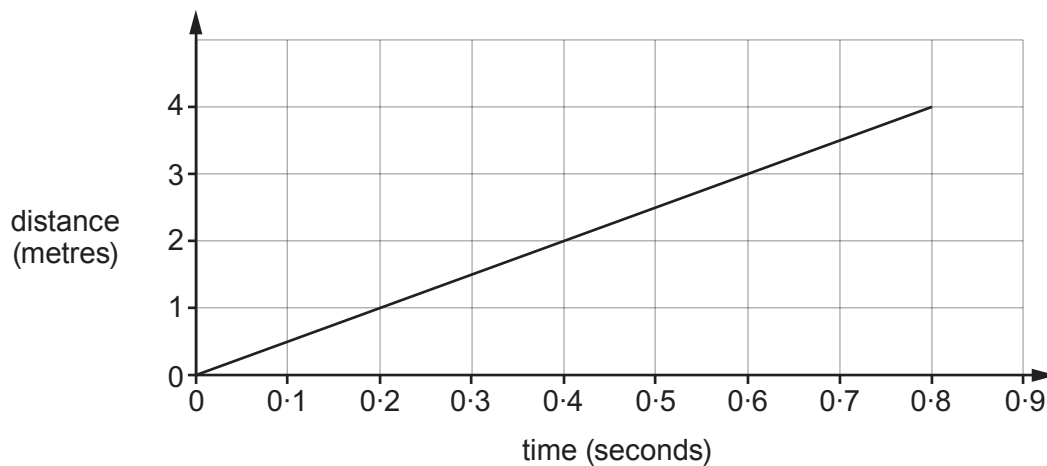
[2]

$$100 = \frac{25}{x}$$

$$100x = 25$$

$$\underline{\underline{x = 0.25}}$$

(b) The distance-time graph shows the start of a journey.



Work out the speed of this part of the journey in metres per second.

[2]

4 m in 0.8 seconds

$$\frac{4}{0.8} = \frac{1}{0.2} = 5$$

5 m/s

7. (a) (i) Simplify $15\pi - \pi$.

[1]

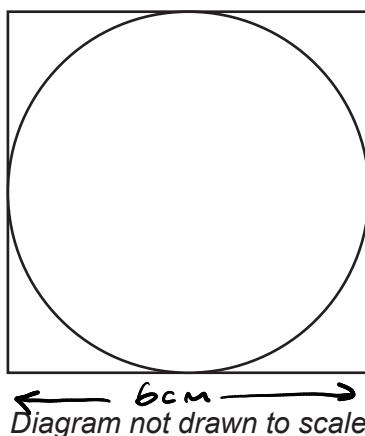
$$14\pi$$

- (ii) Work out $12\pi \div 3\pi$.

[1]

$$4$$

- (b) The diagram shows a circle inside a square.
The circumference of the circle touches all four sides of the square.



The perimeter of the square is 24 cm.

Work out the area of the circle.
Give your answer as a multiple of π .

[3]

$$24 \div 4 = 6\text{ cm}$$

$$r = 3\text{ cm}$$

$$\begin{aligned} \text{Area} &= \pi r^2 \\ &= \pi (3)^2 \\ &= 9\pi \text{ cm}^2 \end{aligned}$$

8. (a) Work out $3\frac{1}{5} - 1\frac{2}{7}$.

[3]

$$\frac{7 \times 16}{7 \times 5} - \frac{9 \times 5}{7 \times 5}$$

$$\frac{112}{35} - \frac{45}{35} = \frac{67}{35} = 1 \frac{32}{35}$$

- (b) Three two-digit integers a , b and c are in the ratios

$$a : b = 4 : 5 \quad \times 7 \quad \text{and} \quad b : c = 7 : 11. \quad \times 5$$

Find the integers a , b and c .

[3]

$$\begin{array}{l} a : b \\ 28 : 35 \end{array} \qquad \begin{array}{l} b : c \\ 35 : 55 \end{array}$$

$$\begin{array}{l} a : b : c \\ 28 : 35 : 55 \end{array}$$

$$a = 28 \qquad b = 35 \qquad c = 55$$

- (c) A length of string has been cut into two pieces in the ratio 3 : 5.
The longer piece measures 205 cm.

What was the original length of the string?

[2]

$$\begin{array}{l} 5 \text{ parts} = 205 \text{ cm} \\ 1 \text{ part} = 41 \text{ cm} \\ 8 \text{ parts} = 41 \times 8 \\ = 328 \text{ cm} \end{array} \qquad \begin{array}{r} 041 \\ 5 \overline{)205} \end{array}$$

9. (a) In a warehouse, 4 workers can load 5 tonnes of goods into a vehicle in 3 hours.

How long would it take 6 workers to load 10 tonnes of goods into a vehicle?
You may assume that all workers work at the same rate.

[3]

4 workers can load 10 tonnes in 6 hrs

$4 \times 6 = 24$ (24 hours for 1 worker)

$24 \div 6 = 4$ 4 hours

- (b) State one other assumption you have made in your answer to part (a).
How would your answer to part (a) change if this assumption were not correct?

[2]

All the goods are the same. If there are more boxes or difficult to carry goods it could take longer.

10. (a) Olly is trying to solve this inequality.

$$10 - 2x > 3$$

Here is Olly's solution.

| | | |
|--------|---------------------|-------------|
| Step 1 | $10 - 2x > 3$ | (-10) |
| Step 2 | $-2x > -7$ | $(\div -2)$ |
| Step 3 | $x > \frac{-7}{-2}$ | |
| Answer | $x > 3.5$ | |

Is Olly's solution correct?
Show clearly how you decide.

[2]

No. Let $x = 4$ $4 > 3.5$

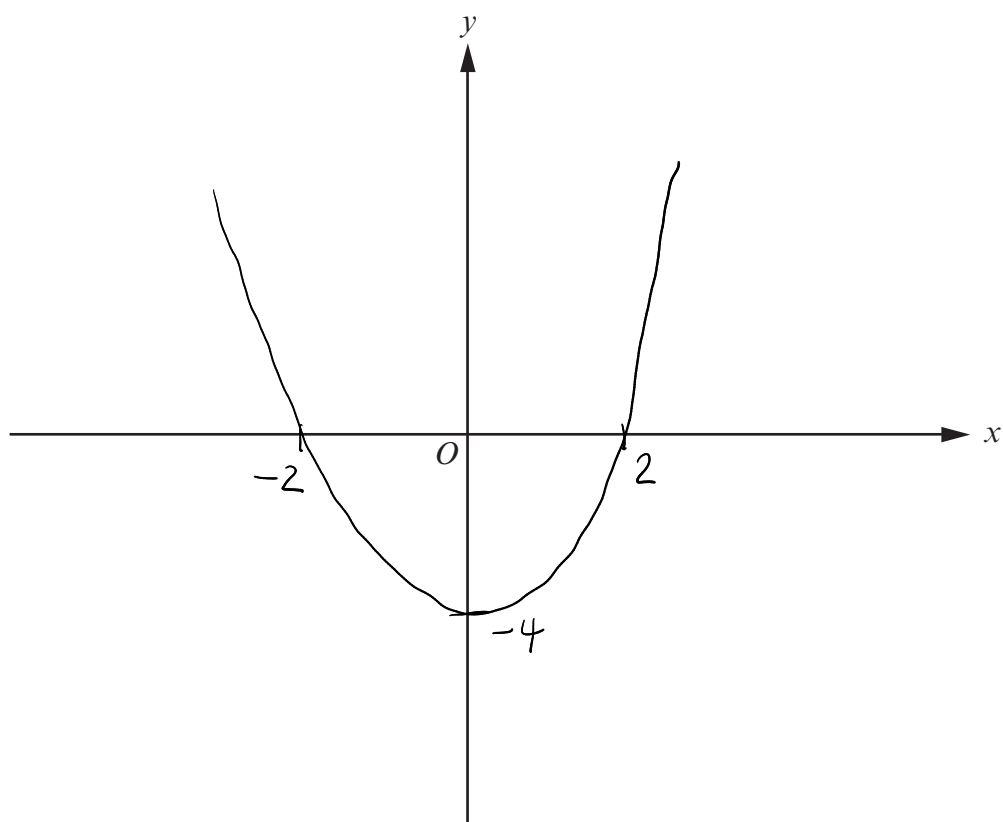
but: $10 - 2(4) = 2$
 $2 > 3$ is not true.

when you \times or \div by a negative the
sign flips. It should be $x < 3.5$

- (b) (i) Sketch the graph of $y = x^2 - 4$ for values of x between -3 and 3 on the axes opposite. Mark any intercepts with the x -axis.

[2]

| | | | | | | | |
|-----|------|------|------|------|------|-----|-----|
| x | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
| y | 5 | 0 | -3 | -4 | -3 | 0 | 5 |



(ii) Solve $x^2 - 4 < 0$.

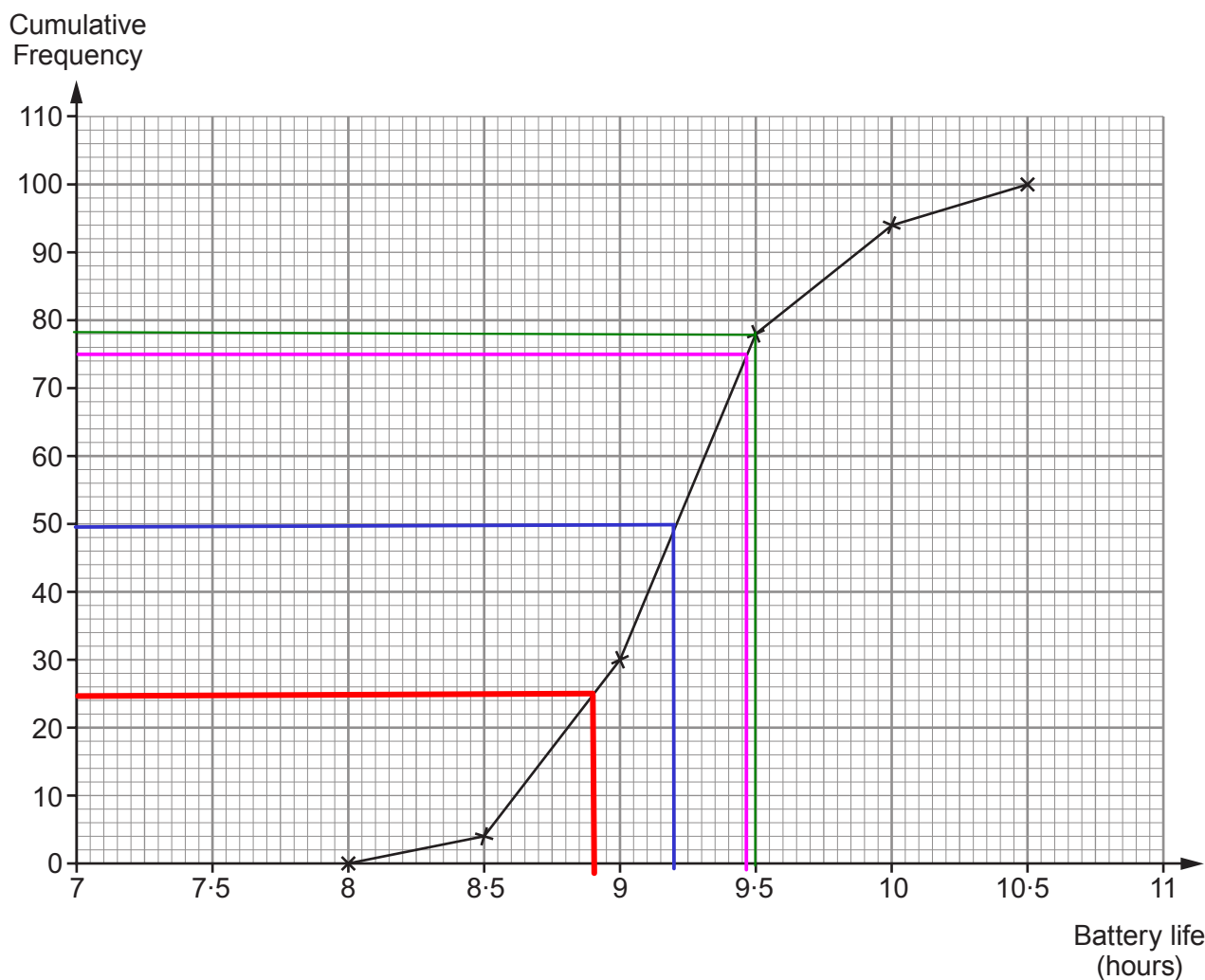
[2]

$$\underline{-2 < x < 2}$$

11. (a) Battery life is the length of time for which a battery can be used before it needs to be recharged.

One hundred users of the *Litestar A* tablet computer each recorded the battery life of their tablet.

The diagram shows the distribution of the battery lives recorded, in hours.



- (i) How many of these *Litestar A* tablet computers had a battery life of more than 9.5 hours? [1]

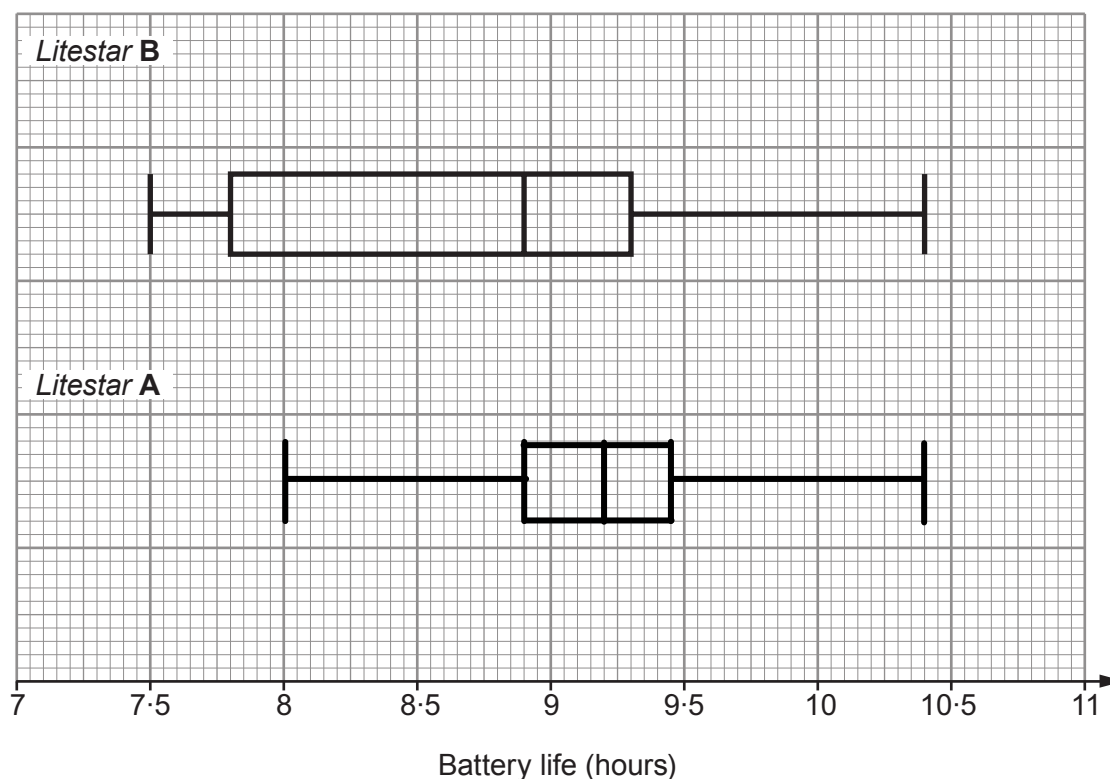
$$100 - 78 = 22$$

- (ii) Use the diagram to complete the table for these *Litestar A* tablet computers. [3]

| Median | Lower Quartile | Upper Quartile | Inter-quartile Range |
|--------|----------------|----------------|----------------------|
| 9.2 | 8.9 | 9.45 | 0.55 |

$$9.45 - 8.9 = 0.55$$

- (b) This box plot shows the distribution of the battery lives recorded by one hundred users of *Litestar B* tablet computers.



- (i) For the *Litestar A* tablet computers:
- The shortest battery life recorded was 8 hours.
 - The longest battery life recorded was 10.4 hours.

Using this information and the information from part (a), draw the box plot for the *Litestar A* tablet computers on the grid above. [2]

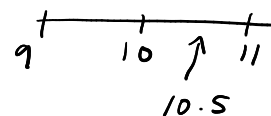
- (ii) Neena wants to buy one of these tablet computers. She wants the best battery life possible.

Should Neena buy a *Litestar A* or *Litestar B* tablet computer?

Litestar A ☒ *Litestar B* ☐

Give a reason for your decision. [1]

The median battery life is longer.

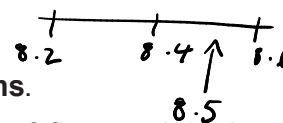
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12. Mahima works in a biscuit factory.

She uses a trolley to move sacks of flour and boxes of chocolate.
It is not safe to have more than 215 kg on the trolley.

Each sack of flour weighs 10 kg, **correct to the nearest kilogram**.

Each box of chocolate weighs 8.4 kg, **correct to the nearest 200 grams**.



Mahima thinks it is always safe to use her trolley to move 4 sacks of flour and 20 boxes of chocolate at the same time.

Is Mahima correct?

Justify your answer with calculations.

[5]

upper bound of flour 10.5 kg

upper bound of choc. 8.5 kg

$$4 \times 10.5 + 20 \times 8.5 = \underline{212.5 \text{ kg}}$$

Yes. Mahima is correct.

13. Rearrange the formula

$$y = \frac{5+x}{w-2x}$$

to make x the subject.

[5]

$$y(w-2x) = 5+x$$

$$wy - 2xy = 5 + x$$

$$wy - 5 = x + 2xy$$

$$wy - 5 = x(1+2y)$$

$$x = \frac{wy - 5}{1 + 2y}$$

14. Simplify $\sqrt[3]{64} \times 2^{-4} \times 4^9$.
Give your answer as a power of 2.

[4]

$$4 = 2^{-4} \times (2^2)^9$$

$$2^2 \times 2^{-4} \times 2^{18}$$

$$\underline{\underline{2^{16}}}$$

15. Fifty people order food and drink for a party.
They each order one main course and one drink from the menu.

| Menu | |
|-------------|-------|
| Main course | Drink |
| Pizza | Cola |
| Burger | Water |
| Kebab | Juice |

28 people order cola.

24 people order a burger.

8 people order a kebab and none of these order cola.

The 5 people who order water all order a kebab.

Of the people who order a burger, twice as many order cola as order juice. $24 \div 3 = 8$

A person is selected at random from the group.

Using the table to help you, find the probability that this person orders either a burger and cola or a pizza and juice.

You must show all your working.

[7]

| | Cola | Water | Juice | Total |
|--------|------|-------|-------|-------|
| Pizza | 12 | 0 | 6 | 18 |
| Burger | 16 | 0 | 8 | 24 |
| Kebab | 0 | 5 | 3 | 8 |
| Total | 28 | 5 | 17 | 50 |

$$\frac{12 + 6}{50} = \frac{22}{50}$$

16. In the diagram,
- ACE is a triangle,
 - $BCDFG$ is a regular pentagon.

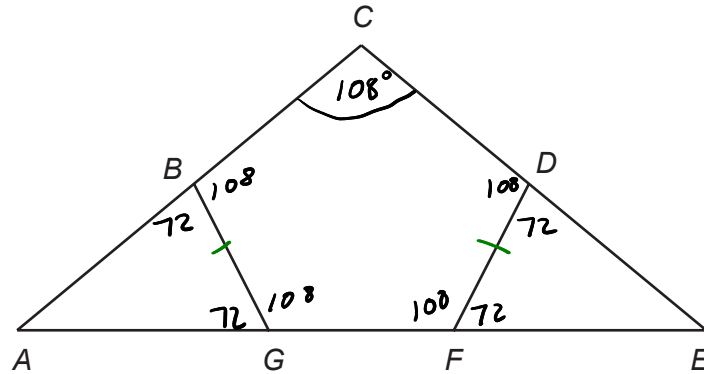


Diagram not drawn to scale

Prove that triangle ABG is congruent to triangle EDF .
Give a reason for each statement you make in your proof.

[5]

Exterior angle in a pentagon $\frac{360}{5} = 72$

$180 - 72 = 108^\circ$ (All interior angles)

$180 - 108 = 72^\circ$ (Angles on a straight line)

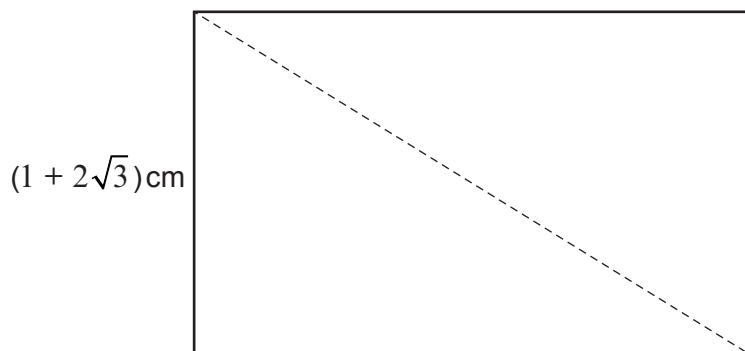
$BG = DF$ as $BCDFG$ is a regular pentagon

$\angle ABG = \angle GAB = \angle EDF = \angle DEF$ all 72°

Conclusion:

ABG is congruent to EDF ASA

17.

*Diagram not drawn to scale*The area of the rectangle is 22 cm^2 .The width of the rectangle is $(1 + 2\sqrt{3}) \text{ cm}$.

- (a) Find the length of the rectangle. Express your answer in the form $a + b\sqrt{3}$, where a and b are integers.

[4]

$$\text{length} = \frac{22}{(1 + 2\sqrt{3})} (1 - 2\sqrt{3})$$

$$= \frac{22 - 44\sqrt{3}}{1 - 2\sqrt{3} + 2\sqrt{3} - 12}$$

$$= \frac{22 - 44\sqrt{3}}{-11}$$

$$= \underline{\underline{-2 + 4\sqrt{3}}}$$

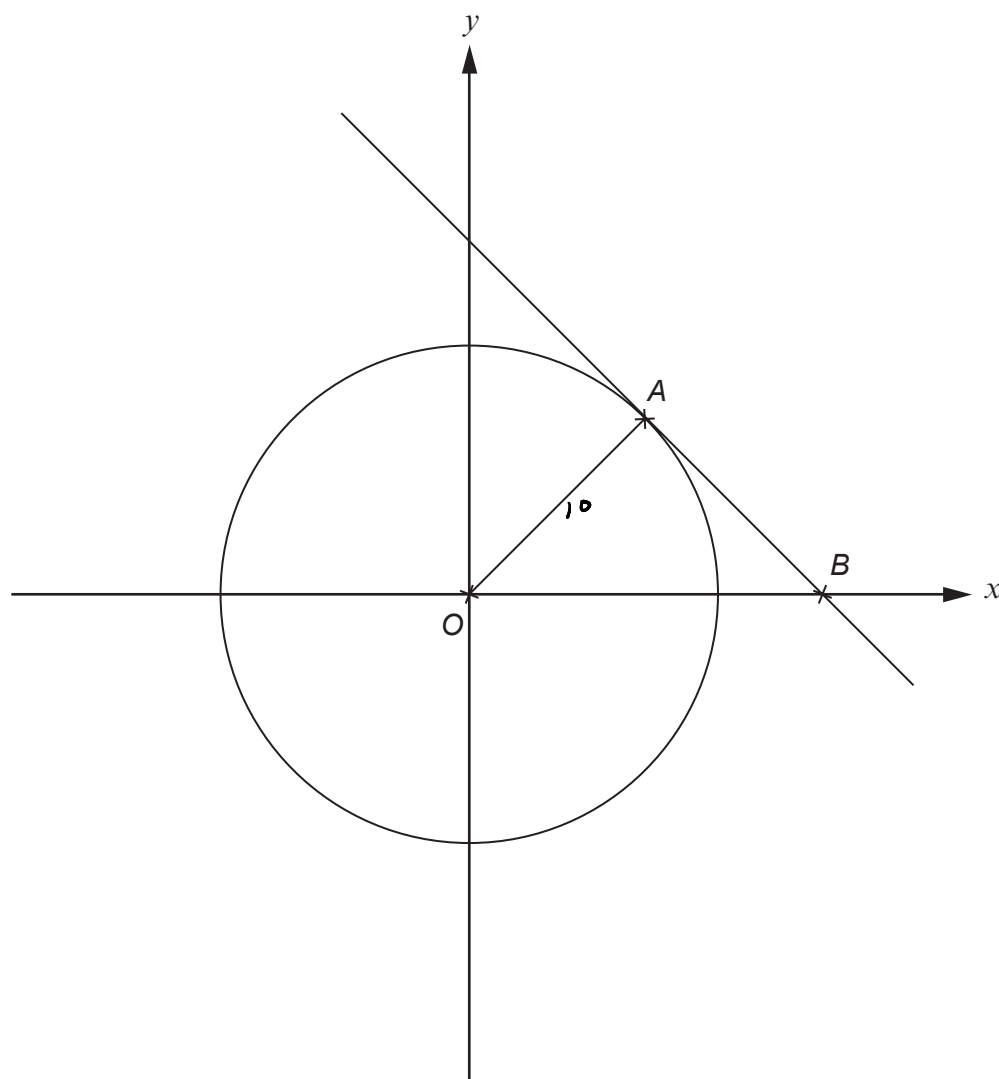
- (b) The length of the diagonal of the rectangle is $\sqrt{x} \text{ cm}$.

Show that $x = (65 - 12\sqrt{3}) \text{ cm}$.

[2]

$$\begin{aligned} x &= (-2 + 4\sqrt{3})^2 + (1 + 2\sqrt{3})^2 \\ &= (-2 + 4\sqrt{3})(-2 + 4\sqrt{3}) + (1 + 2\sqrt{3})(1 + 2\sqrt{3}) \\ &= 4 - 8\sqrt{3} - 8\sqrt{3} + 48 + 1 + 2\sqrt{3} + 2\sqrt{3} + 12 \\ &= \underline{\underline{65 - 12\sqrt{3}}} \end{aligned}$$

18.

*Diagram not drawn to scale*

The diagram shows a circle with centre O.

The gradient of the radius OA is $\frac{4}{3}$.

OA = 10 units.

The tangent to the circle at A meets the x -axis at B.

(a) Explain why the coordinates of A are (6, 8).

[2]

$$x^2 + y^2 = 100 \quad (\text{equation of circle})$$

$$y = \frac{4}{3}x \quad (\text{equation of radius})$$

$$6^2 + 8^2 = 100 \quad \checkmark$$

$$8 = \frac{4}{3}(6) \quad \checkmark$$

(6, 8) lies on the
circle and the radius.

(b) Find the coordinates of B.

[5]

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gradient of tangent $-\frac{3}{4}$ $(6, 8)$
 x y

$$y = -\frac{3}{4}x + c$$

$$8 = -\frac{3}{4}(6) + c$$

$$8 = -\frac{18}{4} + c$$

$$8 = -\frac{9}{2} + c$$

$$c = \frac{25}{2}$$

$$y = -\frac{3}{4}x + \frac{25}{2}$$

crosses x when $y=0$

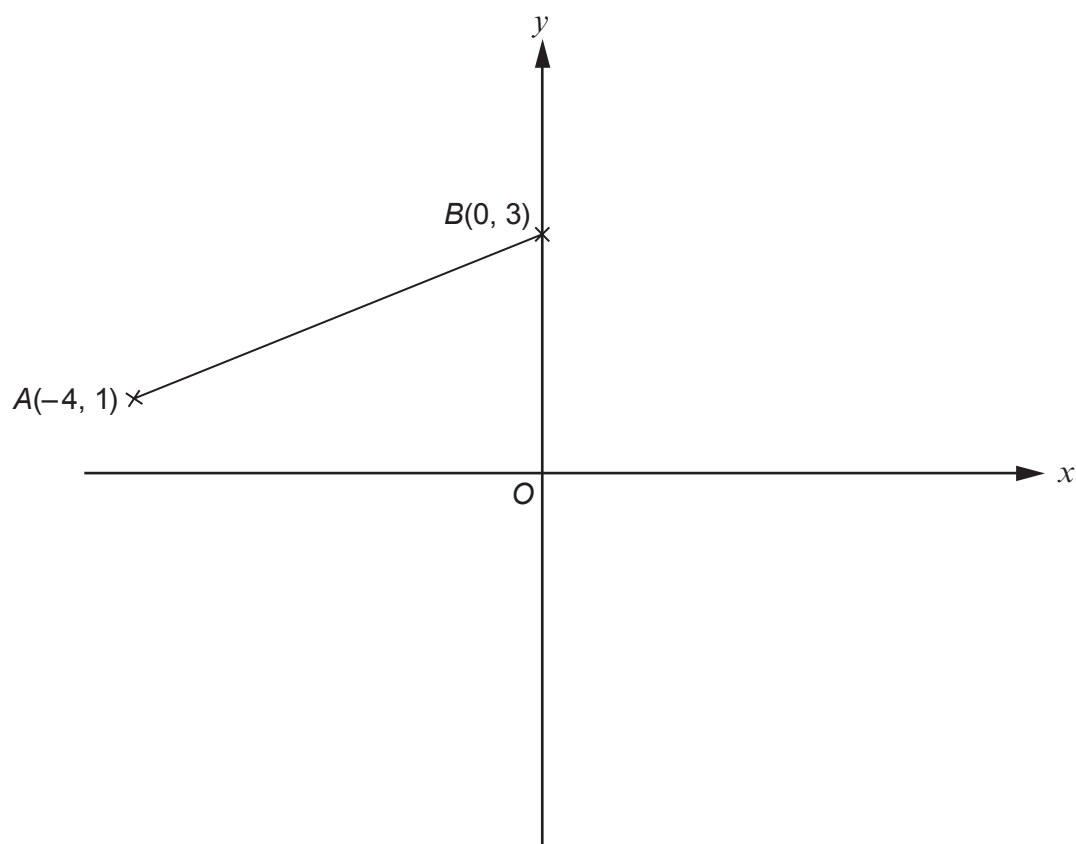
$$0 = -\frac{3}{4}x + \frac{25}{2}$$

$$\frac{3}{4}x = \frac{25}{2}$$

$$x = \frac{100}{6} = \underline{\underline{\frac{50}{3}}}$$

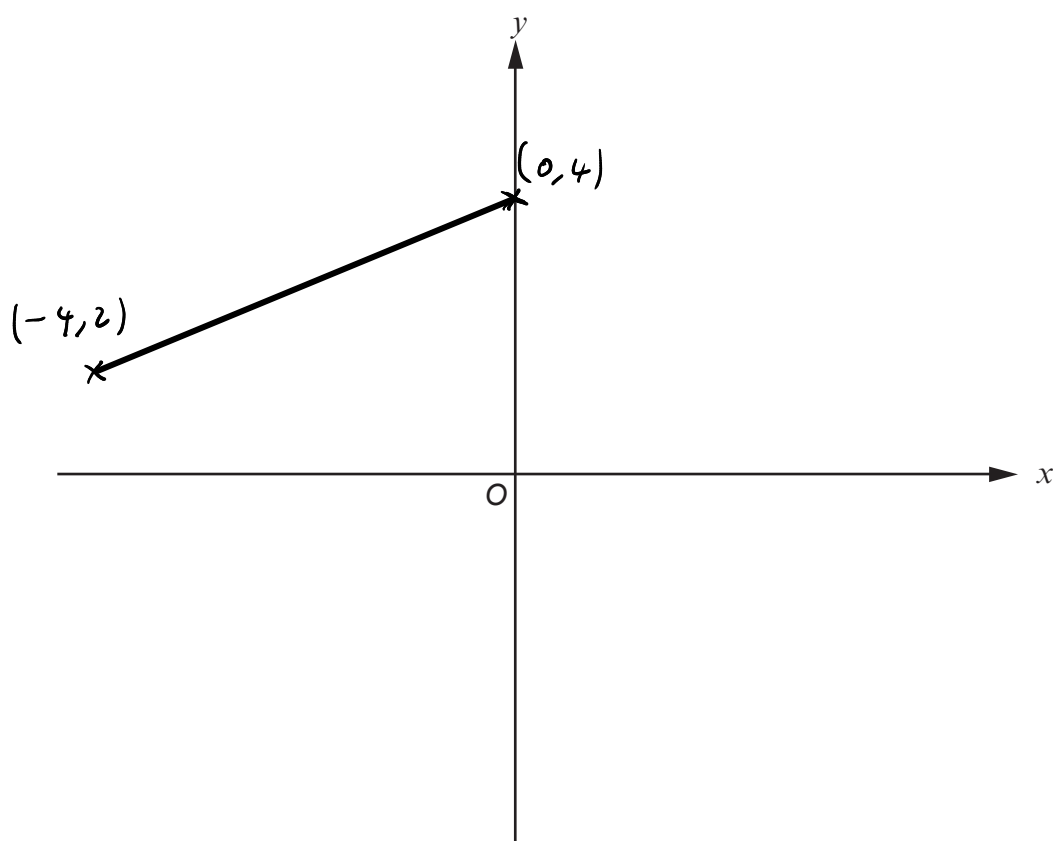
$$\underline{\underline{\left(\frac{50}{3}, 0\right)}}$$

19. The diagram below shows a sketch of the graph of $y = f(x)$.



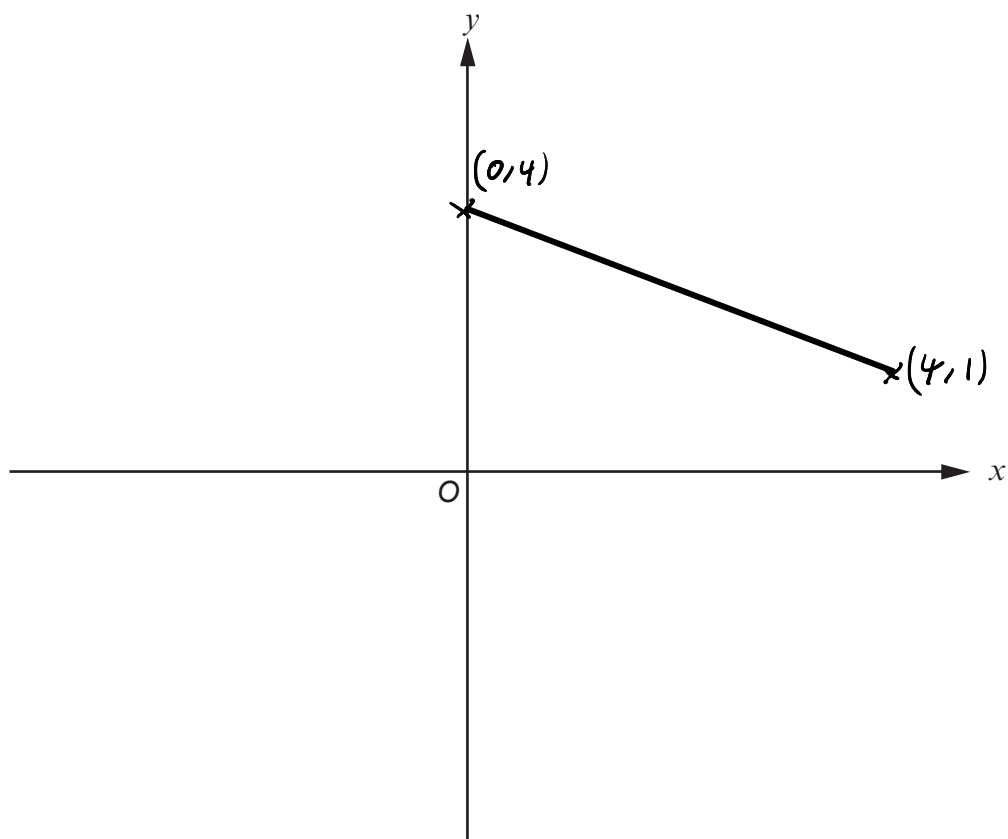
- (a) Sketch the graph of $y = f(x) + 1$ on the axes below.
You must indicate the coordinates of the new positions of the points A and B.

[2]



- (b) Sketch the graph of $y = f(-x)$ on the axes below.
You must indicate the coordinates of the new positions of the points A and B .

[2]



20. (a) (i) How many different 5-digit whole numbers can be made using the digits 2, 3, 4, 5, and 6 when each digit can be used once only? [2]

$$5 \times 4 \times 3 \times 2 \times 1 = \underline{\underline{120}}$$

- (ii) What proportion of the 5-digit whole numbers are odd? [1]

$$\frac{2}{5}$$

- (b) How many different 7-digit **even** whole numbers can be made using the digits 3, 4, 5, 6, 7, 8 and 9 when each digit can be used once only? [2]

$$\frac{3}{7} \text{ even } \frac{3}{7} \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$$

$$18 \times 120$$

$$\underline{\underline{2160}}$$

$$120$$

$$\begin{array}{r} \times 18 \\ 960 \\ 1200 \\ \hline 2160 \end{array}$$

21.

$$f(x) = 5x + 2$$

$$g(x) = x^3$$

$$x = f(10)$$

(a) Solve $f^{-1}(x) = 10$.

[4]

$$5(10) + 2 = 52$$

(b) (i) Show that $gf(x) = 125x^3 + 150x^2 + 60x + 8$.

[3]

$$gf(x) = (5x + 2)^3$$

$$= (5x + 2)(5x + 2)(5x + 2)$$

$$= (25x^2 + 10x + 10x + 4)(5x + 2)$$

$$= (25x^2 + 20x + 4)(5x + 2)$$

$$= 125x^3 + 50x^2 + 100x^2 + 40x + 20x + 8$$

$$= 125x^3 + 150x^2 + 60x + 8$$

(ii) Find $gf(-1)$.

[1]

$$125(-1)^3 + 150(-1)^2 + 60(-1) + 8$$

$$-125 + 150 - 60 + 8$$

$$\underline{\underline{-27}}$$

22. (a) Write the expression $x^2 - 6x + 19$ in the form $(x + a)^2 + b$, where a and b are integers.

[3]

$$(x - 3)^2 - 9 + 19$$

$$(x - 3)^2 + 10$$

- (b) State the coordinates of the turning point of the curve $y = x^2 - 6x + 30$.

[2]

$$(x - 3)^2 - 9 + 30$$

$$(x - 3)^2 + 21$$

Turning point = $(3, 21)$

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